# The Crystallographic Mechanism of the Martensite Reaction in Iron-Carbon Alloys 

By J. S. Bowles*<br>Institute for the Study of Metals, University of Chicago, Chicago, Illinois, U.S.A.

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#### Abstract

Determination of the crystallographic mechanism of a martensite transformation involves the determination of: (1) the total atomic displacements; (2) the paths taken by the atoms during these displacemerits.

Further confirmation of the minimum displacement hypothesis of Jaswon \& Wheeler is given by demonstrating that these displacements produce the lattice-plane transformations determined by Greninger \& Troiano.

These displacements cannot occur as a simple homogeneous distortion since this would not be consistent with the observed relief effects produced by the transformation in iron-carbon alloys. Paths must therefore be ascribed to the atoms to achieve geometrical consistency. Certain assumptions are made which permit these paths to be determined from a stereographic analysis of the Kurdjumow-Sachs orientation relationship, and the conclusion is reached that each atom moves first in the austenite twinning direction and then in the martensite twinning direction. The proposed mechanism is consistent with all the geometrical features of the transformation; good agreement is found between the predicted and observed angles through which polished surfaces are tilted by the production of martensite plates. It is also compatible with a mode of development of martensite plates in which an interface parallel to (225) ${ }_{A}$ migrates into the austenite.


## Introduction

The diffusionless nature of the martensite transformation in steel has led to much speculation concerning the nature of the atomic movements by which the transformation is accomplished, but none of the proposals which have been made so far is fully satisfactory.

To be satisfactory, a proposed crystallographic mechanism must be consistent with the following geometrical features of the transformation:
(1) The orientation relationship between austenite and martensite.
(2) The martensite habit plane.
(3) The angles through which polished surfaces are tilted by the production of martensite plates.
(4) The angles through which scratches on polished surfaces are bent by the production of martensite plates.

The orientation relationship between austenite and martensite varies with the composition of the austenite, and the reported relationships cover the range between two simple extremes. These extremes are the well-known Kurdjumow-Sachs and Nishiyama relationships which can be described respectively by $\dagger$

$$
\begin{array}{lll}
(111)_{A} \|(101)_{M} & \text { and } & (111)_{A} \|(101)_{M} \\
{[1 \overline{1} 0]_{A} \|[11 \overline{1}]_{M}} & & {[1 \overline{2} 1]_{A} \|[10 \overline{1}]_{M}}
\end{array}
$$

[^0]The Kurdjumow-Sachs orientation can be produced from the Nishiyama by a rotation of $5^{\circ} 16^{\prime}$ about $[101]_{M}$.

The results of two independent studies by Kurdjumow \& Sachs (1930) and by Wassermann (1935) indicate that in $1.4 \%$ carbon steel the Kurdjumow-Sachs relation exists. In iron-nickel alloys containing about $30 \%$ nickel, both Nishiyama (1934) and Wassermann (1935) have found the Nishiyama relationship. Mehl \& Derge (1937), however, report that the orientation relationship in iron-nickel alloys depends on the temperature of formation of the martensite, the Kurdjumow-Sachs relation being produced by a reaction occurring at $240^{\circ}$ C., and the Nishiyama by a low-temperature reaction ( $-195^{\circ}$ C.).

Using a precision technique, Greninger \& Troiano (1941, 1949) have found that in an iron-base alloy containing $22 \%$ nickel and $0.8 \%$ carbon, the orientation relationship is intermediate between the two extremes. Young (1939) also reports various intermediate relationships for the ferrite (kamacite) plates in meteorites.

The martensite habit planes have been investigated by Mehl, Barrett \& Smith (1933) and by Greninger \& Troiano (1940). Greninger \& Troiano report that in iron-carbon alloys containing between 0.55 and $1.4 \%$ carbon the martensite habit plane is $\{225\}_{A}$. In highcarbon alloys the martensite crystals are true plates but for carbon contents below the eutectoid they degenerate into laths which have their long dimension parallel to the $\langle 110\rangle_{A}$ direction. In low-carbon steels
the mariensite appears as 'a plate-like array of striations' parallel to $\{111\}_{\boldsymbol{A}}$. This could be caused merely by a further degeneration of the laths. Mehl, Barrett \& Smith also report a $\{111\}_{A}$ 'habit plane' in low-carbon steel.

In iron-carbon alloys containing $1.78 \%$ carbon (Greninger \& Troiano, 1940), in iron-nickel alloys containing $32.5 \%$ nickel (Greninger \& Troiano, 1940), and in the iron-nickel-carbon alloy mentioned previously (Greninger \& Troiano, 1941, 1949), the habit plane can be described approximately by the indices $\{259\}_{A}$, but there is a wide degree of scatter.

The first attempts to describe the mechanism of the transformation were made by Bain (1924), Kurdjumow \& Sachs, and Nishiyama. Bain considered only the change in structure, and his mechanism is not consistent with any of the geometrical features of the transformation. Kurdjumow \& Sachs and Nishiyama a.ttempted to account for the observed orientation relationships, but the mechanisms that they proposed are not consistent with the other geometrical features of the transformation, in particular the habit planes.

More recently there have been two important contributions, namely, those of Greninger \& Troiano (1941, 1949), and Jaswon \& Wheeler (1948).

Working with an iron-base alloy containing $0.8 \%$ carbon and $22 \%$ nickel, Greninger \& Troiano analyzed the relief effects produced on a polished surface by the transformation. Assuming that the relief was produced by a shear on the habit plane, they were able to determine the shear angle and shear direction from the angles through which different surfaces were tilted by the production of martensite plates. On applying this experimentally determined shear to austenite, they found that it did not accomplish the transformation. In order to complete the transformation a secord shear on the $(112)_{M}$ plane in the $[11 \overline{1}]_{M}$ direction was necessary. These shear elements are the twinning elements in martensite. Greninger \& Troiano therefore proposed that the transformation occurs in two stages, the first stage being a homogeneous shear and producing the observed relief effects, the second being a shear, homogeneous within narrow lamellae (not less than 18 atomic planes in thickness), but macroscopically heterogeneous and causing no observable change in the shape of the plate.

This mechanism explains the habit plane as being the plane of the first shear. It is consistent with the observed relief effects, and gives approximately the correct orientation relationship. The predicted martensite dimensions are, however, too small and it is necessary to postulate that an expansion occurs either before, after or during the shears. This expansion amounts to as much as $\mathbf{4 \cdot 2} \%$ in the $[100]_{M}$ direction.

There can be no denying that this mechanism goes a long way towards accounting for all the geometrical features of the transformation in this iron-nickelcarbon alloy. However, it is not consistent with the
geometrical features of the transformation in other compositions, e.g., plain carbon steel containing less than $1.4 \%$ carbon. In this case it is not possible to account for the Kurdjumow-Sachs relationship by a shear on the habit plane, which in this case is $\{225\}_{A}$, followed by a shear on the martensite twinning elements.

It should be noted that Greninger \& Troiano's assumption of shear was not justified quantitatively by their experimental results, which indicate a scatter of some $30^{\circ}$ in the shear direction. This may be a result of the very difficult experimental technique, or it may indicate that the martensite relief cannot be attributed to a shear on the habit plane.


Fig. 1. Stereographic projection illustrating the KurdjumowSachs relationship. The hollow symbols are austenite poles, the filled symbols martensite poles. The variant shown is the standard variant.

Jaswon \& Wheeler have attempted, with some success, to identify the martensite habit plane with one of the crystallographic planes that do not rotate during the transformation. Any rotation of the habit plane during transformation would obviously be energetically unfavorable. In their treatment, which considers only the Kurdjumow-Sachs relationship, ${ }^{*}$ Jaswon \& Wheeler assumed that during the transformation each atom in the austenite moves to the nearest available position in the martensite structure. Treating these minimum displacements as a homogeneous finite strain they derived a strain matrix from which they were able to determine the planes which are not rotated by the strain. Three such planes were found. For the standard variant one of these planes is $(111)_{A}$, another is a plane

[^1]which lies within $1^{\circ}$ of $(111)_{\mathcal{A}}$ and becomes coincident with it for zero carbon content, and the third is a plane which for all carbon contents lies within $1.5^{\circ}$ of $(225)_{A}$. $*$

It should be noted that this result means that each variant of the Kurdjumow-Sachs relationship has a specific $\{225\}_{A}$ plane as habit plane. Jaswon \& Wheeler have predicted that the standard variant will have the $(225)_{A}$ plane as habit plane, and not any other plane of the form $\{225\}_{\boldsymbol{A}}$. There is no direct experimental evidence on this point.

It is also shown in this analysis that the tetragonality of martensite can be explained if it is assumed that the displacements of the iron and carbon atoms constitute a common homogeneous distortion.

Jaswon \& Wheeler claim that there is no experimental evidence on which a theory of the paths taken by the atoms during their displacements can be based. This will be true only if it is possible to account for all the geometrical features of the transformation by assuming that the atom displacements occur as a simple homogeneous distortion. If it is not possible to do this, then these geometrical features form an important base upon which a theory of the atom paths can be built.

## Proposed mechanism

The determination of the crystallographic mechanism of the martensite transformation involves two major problems:
(1) The determination of the total atomic displacements which occur.
(2) The determination of the paths taken by the atoms during these displacements.

## (1) Atomic displacements

If the transformation strain is homogeneous, then the atomic displacements, when known, can be specified most simply by a relation between the co-ordinate numbers of the atoms before and after the transformation, i.e. by a relation of the type

$$
\left(\begin{array}{l}
x^{\prime} \\
y^{\prime} \\
z^{\prime}
\end{array}\right)_{M}=(\tau)\left(\begin{array}{l}
x \\
y \\
z
\end{array}\right)_{A}
$$

where $(x y z)_{\mathcal{A}}$ are the co-ordinate numbers, referred to austenite axes, of the atoms before transformation, $\left(x^{\prime} y^{\prime} z^{\prime}\right)_{M}$ are the co-ordinate numbers, referred to martensite axes, of the atoms after transformation, and $(\tau)$ is the matrix of the linear equations relating these co-ordinates. Such a relation, when combined with the orientation relationship and the dimensions of the two lattices, provides a complete description of the atomic displacements within homogeneously strained regions.

Jaswon \& Wheeler have proposed that the displacements which seem most likely to occur are those which

[^2]move each atom in austenite to the nearest available position in the martensite structure. For the standard variant of the Kurdjumow-Sachs relationship, the coordinate transformation matrix ( $\tau$ ) describing these minimum displacements (Jaswon \& Wheeler, 1948) is
\[

(\tau)=\left($$
\begin{array}{lll}
1 & 0 & 1  \tag{1}\\
1 & 0 & \bar{l} \\
0 & 1 & 0
\end{array}
$$\right)
\]

The success of the Jaswon \& Wheeler analysis in predicting the $(225)_{A}$ habit plane and in explaining the tetragonality of martensite provides some confirmatory evidence for this minimum net displacement hypothesis. Further confirmation is supplied by consideration of the plane transformations. The above atomic-co-ordinate transformations produce a corresponding set of plane transformations which can be determined as follows:
The equation of a plane $(h k l)_{A}$ in austenite is

$$
\begin{equation*}
h x+k y+l z=1 \tag{2}
\end{equation*}
$$

This plane transforms into the plane $\left(h^{\prime} k^{\prime} l^{\prime}\right)_{M}$, whose equation is

$$
\begin{equation*}
h^{\prime} x^{\prime}+k y^{\prime}+l^{\prime} z^{\prime}=1 \tag{3}
\end{equation*}
$$

where $(x y z)_{A}$ and $\left(x^{\prime} y^{\prime} z^{\prime}\right)_{M}$ are related by the coordinate transformation matrix (1). On substituting in (3) the values of ( $x^{\prime} y^{\prime} z^{\prime}$ ) given by (1), and collecting terms, one obtains
$\left(1 h^{\prime}+1 k^{\prime}+0 l^{\prime}\right) x+\left(0 h^{\prime}+0 k^{\prime}+l^{\prime}\right) y+\left(1 h^{\prime}+\overline{1} k^{\prime}+0 l^{\prime}\right) z=1$.
Comparison with (2) shows that

$$
\left(\begin{array}{l}
h \\
k \\
l
\end{array}\right)_{A}=\left(\begin{array}{lll}
1 & 1 & 0 \\
0 & 0 & 1 \\
1 & \overline{1} & 0
\end{array}\right)\left(\begin{array}{l}
h^{\prime} \\
k^{\prime} \\
l^{\prime}
\end{array}\right)_{M}
$$

which by inversion gives

$$
\left(\begin{array}{l}
h^{\prime} \\
k^{\prime} \\
l^{\prime}
\end{array}\right)_{M}=\left(\begin{array}{lll}
1 & 0 & 1 \\
1 & 0 & \overline{1} \\
0 & 2 & 0
\end{array}\right)\left(\begin{array}{l}
h \\
k \\
l
\end{array}\right)_{A}
$$

The plane transformations described by this matrix are the same as those deduced by Greninger \& Troiano (1949, fig. 6) from their study of the martensite relief. (Repetition of the Jaswon \& Wheeler analysis for the Nishiyama, and for intermediate relationships, has shown that the co-ordinate transformation matrix corresponding to minimum displacement is the same in all cases.)

If, on the basis of this evidence, we can conclude that the minimum-displacement hypothesis is correct, the problem of determining the atom paths is greatly simplified. The co-ordinate and plane transformation matrices given above allow one to predict the final position after transformation of my austenite direction or plane, respectively. These can then be compared directly with the apparent final positions indicated by the bending of scratches and the tilting of surfaces. This eliminates the necessity for using Greninger \& Troiano's difficult technique of measuring the angles of tilt produced by a single plate on two different surfaces.

## (2) Atom paths

As there is no a priori reason to anticipate that the transformation distortion will be a shear, or two shears, it is of interest to attempt to discover the reason for the success of the Greninger-Troiano theory. If the atoms do not take direct paths from initial to final positions (and there is as yet no experimental evidence on this point for iron-carbon alloys containing less than $1.4 \%$ carbon), the reason is presumably because potential barriers make these straight-line paths impossible. In such a case, since the potential field around every atom is identical, it is to be expected that every atom will move in the same direction. This will be the direction in which the potential barrier is lowest or has vanished (cf. Zener, 1948, chap. 4). The shear is only a special case of such a distortion in which every particle moves in the same direction. In the case of shear the direction of motion is a direction lying in the plane that is not distorted and not rotated by the distortion, i.e. the shear plane. However, in principle this need not be so; the direction of motion need not lie in the undistorted plane. It is proposed therefore that Greninger \& Troiano imposed an unjustified restriction when they analyzed the martensite relief as a shcar.

The behavior that is anticipated is one in which each atom will move first in that direction in austenite in which movement can occur most easily. At a certain stage in the distortion the potential field will have become so changed that this is no longer the direction of easiest motion. The same behavior is to be anticipated in the reverse transformation. Thus, if the atom patins can really be resolved into two components, it is to be expected that the final direction in which the atoms find themselves moving will be the direction of easiest motion in martensite. One should therefore attempt to resolve the total transformation strain into two homogeneous distortions, each of which is characterized by a single direction of motion of atoms and an undistorted plane. For this purpose it is instructive to analyze the stereographic properties of such distortions.

These distortions possess stereographic properties analogous to those which Greringer \& Troiano (1941, 1949) have described for pure shear. In their more general form these properties may be summarized as follows:
(1) Directions, i.e. rows of atoms, lying in the undistorted plane are not affected by the distortion. All other directions move along greai circles containing the poles of the original direction and the direction of atom movement (Fig. 2).
(2) Planes containing the direction of atom movement are not rotated by the distortion and hence the poles of these planes are not shifted. The poles of all other planes move along great circles containing the original pole and the pole of the undistorted plane (Fig. 3).

If two such distortions occur consecutively, certain relations must exist between the original and final
lattice orientations. These relations arise in the following way. During the first distortion, planes containing the direction of atom movement are not shifted, and


Fig. 2. Movement of crystallographic directions, i. $\theta$. rows of atoms, during distortions of the type proposed. The original direction $D_{I}$ and the final direction $D_{F}$ lie on a great circle containing the direction of motion of atoms. Directions in the undistorted plane are not affected by the distortion. The arrows indicate the manner in which various directions are shifted.


Fig. 3. Movement of planes during distortions of the type proposed. The plane $P_{I}$ contains the direction $D$ which is not affected by the distortion since it lies in the undistorted plane. The only possible way that the plane $P_{I}$ can be shifted is by a rotation about $D$. This means that the original pole $P_{I}$ and the final pole $P_{F}$ inust lie on a great circle containing the pole of the undistorted plane.
therefore such planes must move to their final positions purely as a result of the second distortion. As a consequence of this, the great circles through the initial
and final positions of the poles of this zone of planes must all intersect at the pole of the undistorted plane of the second distortion.

Since the final position of any plane is given by the plane transformation matrix and the orientation relation, it is possible to locate on the pole figure a zone satisfying these requirements and thus to determine the first direction of atom movement, and the undistorted plane of the second distortion.


Fig. 4. Stereographic analysis of plane transformations. The hollow symbols are austenite poles, the filled symbols martensite poles. The initial and final positions of the poles of planes containing [ $\Pi 12]_{A}$ lie on great circles which have a common intersection at the pole of (112) $M$. The first direction of movement is thus [112] $]_{A}$ and the second undistorted plane is $(112)_{M}$. The initial and final positions of poles of planes containing [111] $]_{M}$ all lie on the same great circle. The 'intersection' in this case could be either (225) $A$ or (111) $)_{A}$, since neither of these poles is shifted. Thus the second direction of movement is $[11 \overline{1}]_{M}$, and the first undistorted plane is either (225) ${ }_{A}$ or (1111) . The latter possi- $^{\text {. }}$ bility is eliminated since this would mean that the first distortion was a shear and (225) $)_{A}$ should have been moved.

Similarly, planes in the zone of the second direction of atom movement are not rotated by the second distortion and thus their poles must have moved to their final positions purely as a result of the first distortion. Great circles through the initial and final positions of these poles have a common intersection at the pole of the undistorted plane of the first distortion. Thus, by locating this zone, the second direction of movement and the first undistcrted plane can be determined.

The directions of movement and the undistorted planes can also be determined by making use of the coordinate transformation matrix which describes the direction transformations, and analyzing the movement of directions during consecutive distortions. In this case the reasoning is as follows. Any direction lying in the undistorted plane of the first distortion is
not affected by this distortion and therefore moves to its final position purely as a result of the second distortion. Therefore the initial and final positions of these directions lie on great circles all of which intersect at the pole of the second direction of atom movement. Similarly, directions lying in the undistorted plane of the second distortion have reached their final positions by the first distortion, and therefore the initial and final positions of these directions lie on great circles which


Fig. 5. Stereographic analysis of direction transformations. The hollow symbols are austenite directions, the filled symbols martensite directions. The initial and final positions of directions, i.e. rows of atoms, lying in the (112) ${ }_{M}$ plane lie on great circles which have a cominon intersection at [112] . These rows of atoms have moved to their final positions as a result of the first distortion. The initial and final positions of directions lying in the $(225)_{A}$ plane all lie in the (225) $A_{A}$ plane. The only direction in this plane that is not moved is the $[11 \overline{1}]_{M}$ direction. These directions are not moved by the first distortion because they lie in the undistorted plane; they are moved to their final positions by the second distortion.
intersect each other at the pole of the first direction of atom movement. These principles have been applied to the standard variant of the Kurdjumow-Sachs relationship and the results obtained for the case of pure iron, i.e. body-centered cubic 'martensite', are shown in Table 1 and in Figs. 4 and 5. In making this stereographic analysis one uses Jaswon \& Wheeler's result

## Table 1

| Direction <br> of atom <br> movement | Undistorted |
| :---: | :---: |
| $[\overline{112} 2]_{M}$ | plane |
| $[111]_{M}$ | $(225)_{\Delta} *$ |
| $(112)_{M}$ |  |

$\stackrel{\text { First distortion }}{\text { Second distortion }}$

* The first undistorted plane is the same as that found by Jaswon \& Wheeler, i.e. $(2,2,4 \cdot 9)_{A}$ which is $0.5^{\circ}$ from $(225)_{A}$. In the interests of simplicity this plane, which varies slightly with the carbon content, is referred to throughout as the (225) 4 plane.
that the $(225)_{\boldsymbol{A}}$ and the $(111)_{A}$ planes are not rotated by the transformation. Although it has not been proven rigorously that the solution given in Table 1 is the only possible solution, it is the only one which a systematic search has revealed.

In connection with the results of this analysis, the following points will be noted:
(1) The first distortion is not a shear; the $[\overline{1} \overline{1} 2]_{A}$ direction does not lie in the $(225)_{A}$ plane.
(2) The second distortion predicted by this analysis is the same as that proposed by Greninger \& Troiano.
(3) On the hypothesis that the habit plane is the undistorted plane of the first distortion, the analysis predicts the correct indices for this plane. The $(225)_{A}$ plane is the undistorted plane of the first distortion, and contains the direction of atom movement in the second. It is therefore not rotated in either case. Although there is no experimental proof that the standard variant has the habit plane $(225)_{A}$, and not any other plane with indices 225 (referred to austenite), this prediction receives further support from the fact that the long dimension of the martensite laths in medium-carbon steel is parallel to $\langle\mathrm{ll} 10\rangle_{A}$. For the production of a lath or needle, the essential point would seem to be that the long axis should not be rotated by the transformation. The only direction which is not rotated by the transformation is the intersection of the two undistorted planes, $(225)_{A}$ and $(111)_{A}$. These planes intersect along the direction $[1 \overline{1} 0]_{A}$.
(4) The $(111)_{A}$ plane which transforms into, and is parallel to, the $(101)_{M}$ plane is not rotated by either distortion, since it contains both directions of atom movement.
(5) The analysis gives as directions of movement the directions in which atoms move during twinning in face-centered and body-centered cubic structures, respectively. Since the $\langle 112\rangle_{A}$ direction is the direction in which atoms can move most easily in stable austenite, it is perhaps not surprising that, the potential barrier in this direction would be the first to disappear on cooling.

Figs. 4 and 5 depict the plane and direction transformations in pure iron, i.e. body-centered cubic martensite. For the production of body-centered tetragonal martensite,* the directions of motion and the undistorted plares are the same to within the accuracy of this graphical method. It is the magnitudes of the displacements which differ, the displacements involved in both the first and second distortions being smaller for the production of tetragonal martensite.

This description of the production of tetragonal mar'censite is an excellent approximation, but is not strictly accurate. The inaccuracy can be recognized from the fact that the distortions described above leave only two planes unrotated, whereas Jaswon \& Wheeler have shown that if the orientation relationship is exactly Kurdjumow-Sachs then in the production of tetragonal

[^3]martensite there is a third unrotated plane lying within $1^{\circ}$ of $(111)_{\boldsymbol{A}}$. It is impossible to produce tetragonal martensite in exact Kurdjumow-Sachs relationship to austenite by two consecutive distortions of the type proposed. However, tetragonal martensite certainly can be produced from austenite by two such distortions and it is possible to calculate what the orientation relationship would be. In making a calculation to show this, it has been assumed that the close-packed directions $[1 \overline{1} 0]_{A}$ and $[11 \overline{1}]_{M}$ remain parallel, and that the second distortion is the same as that which occurs in pure iron, and in the iron-nickel-carbon alloy studied by Greninger \& Troiano, i.e. a shear on the martensite twinning elements. This means that the first distortion has to generate the $(112)_{M}$ planes. The calculated orientation is very close to the Kurdjumow-Sachs orientation and can be produced from it by a rotation which varies from zero to $15^{\prime}$, as the carbon content is increased from zero to $1.35 \%(c / a=1.06)$. The rotation axis is the closepacked $[1 \overline{1} 0]_{A} \|[11 \overline{1}]_{M}$ direction.

The undistorted plane of the first distortion varies from $(2.2 .4 \cdot 9)_{A}$ which is $-0.5^{\circ}$ from $(225)_{A}$, to $(2,2,5 \cdot 1)_{A}$, which is $+0.5^{\circ}$ from $(225)_{A}$, as the carbon content is increased to $1.35 \%$. This is even better agreement with the experimentally determined habit plane than that found by Jaswon \& Wheeler. The corresponding angles in the Jaswon \& Wheeler analysis are $-0.5^{\circ}$ from $(225)_{A}$ for zero carbon to $-1.5^{\circ}$ from $(225)_{A}$ for $1.7 \%$ carbon.

The direction of motion in the first distortion varies from $[\overline{11} 2]_{A}$ for zero carbon to $[1.1,2 \cdot 26]_{A}$ for $1.35 \%$ carbon. The plane which remains unrotated by virtue of its containing both directions of movement varies from $(111)_{A}$ to $(1,1,0 \cdot 88)_{A}$ which is $3 \cdot 3^{\circ}$ from $(111)_{A}$.

The proposed mechanism is consistent with the orientation relationship and the habit plane, and the directions of movement of the atoms seem to be capable of a simple physical interpretation. It remains to establish that the mechanism is consistent with the other geometrical features of the transformation.

## Relief effects

The relief effects produced by the transformation consist essentially of a simple tilting of the surface about its intersection with the habit plane; the line of intersection is not rotated. To produce this kind of relief the habit plane must be a plane of zero macroscopic distortion, i.e. there can be no macroscopic rotation of the habit plane, nor can there be any macroscopic rotation of rows of atoms within this plane. This condition is satisfied if it is proposed (ef. Greninger \& Troiano) that the second shear occurs heterogeneously* and causes no observable change in the shape of the plate. The relief effects would then be produced entirely by the first

* Because of this heterogeneity, the atomic-co-ordinate transformation matrix applies only within the regions which have been homogeneously strained. However, if the heterogeneity is on a scale larger than the unit cell, this same matrix can still be used to describe the direction transformations that gccur in the crystal as a whole.
distortion and since this distortion is characterized by the undistorted plane, $(225)_{A}$, they would be of exactly the type observed, i.e. a tilting of the surface about its intersection with the $(225)_{A}$ plane. The $(111)_{A}$ plane is not a permissible habit plane for, although it is an unrotated plane, rows of atoms within it are rotated during the first distortion. The intersection of the $(111)_{A}$ plane and the surface would therefore be rotated. The proposed mechanism thus provides an explanation for the choice of the (225) ${ }_{A}$ plane as habit plane, in preference to the (111) ${ }_{A}$ plane. This choice cannot be explained if zero rotation alone is regarded as the criterion governing the selection of a habit plane.

The analysis that has been made of the transformation distortions makes it possible to predict the angle of tilt of any surface and to compare the predicted angles with measured angles. For this purpose it is necessary to know, for every martensite plate investigated, the austenite orientation and the martensite orientation. Since it is impracticable to make orientation determinations on single martensite plates in iron-carbon alloys, it is necessary to resort to the less direct technique of plotting the martensite orientation from the known orientation relationship and the obserred habit plane. To do this it has been assumed that the habit plane of the standard variant of the Kurdjumow-Sachs relationship is $(225)_{A}$, and that in other variants the habit plane always bears the same relationship to the martensite lattice as it does in this case. Unless this assumption is correct, the proposed mechanism could not possibly be correct.

A further difficulty in plotting the martensite orientation arises from the fact that whereas there are 24 variants of the Kurdjumow-Sachs relationship, the $\{225\}_{A}$ habit plane only has a multiplicity of 12 . This discrepancy between the multiplicity of the orientation relationship and that of the habit plane can only mean that each $\{225\}_{A}$ plane is used as habit plane by two different variants of the orientation relationship, i.e. a given plate can have either of two different orientations. This can only be the case if two of the variants bear exactly the same relationship to a given $\{225\}_{A}$ plane.

The proposed relationship between the habit plane and the martensite lattice receives further support from the fact that there are two variants of the KurdjumowSachs relation which bear exactly the same relationship to the proposed habit plane. These are illustrated in Fig. 6. The two variants are in the twin relationship to each other and the $(7 \overline{3} 4)_{M} \|(225)_{A}$ plane is common to both. Also shown in Fig. 6 are the movements of planes by which it is proposed that these two variants are produced. It will be noted that the first distortion is identical in the two cases, and the second distortions differ only with respect to the sense of the shear. The reason for this behavior is that the first distortion generates the $(112)_{M}$ plane which is the same in both cases since the two variants are twins.

In view of this result it is now clear that as far as the analysis of angles of tilt is concerned, it is not necessary to be able to distinguish between the two martensite orientations which a given plate can possess; the predicted angle of tilt of any surface would be the same in both cases. It is only necessary to know which $\{225\}_{A}$ plane is being utilized as habit plane to predict the angle of tilt. It can be seen from Fig. 6 that if the displacements occurred as a single homogeneous distortion, the angles of tilt produced on any surface by the formation


Fig. 6. Stereographic projection showing austenite in standard projection (crosses), the stendard variant of the Kurd-jumow-Sachs relationship (hollow symbols), and its twin on the (112) ${ }_{M}$ plane (filled symbols). Both orientations bear the same relationship to the (225) plane. The curves indicate the movement of planes by which these two variants are produced. The first distortion is the same in both cases and the second distortions differ only with respect to the sense of the shear.
of these twin martensite orientations would not be equal, nor would the tilts be in the same sense. One can therefore test the Jaswon \& Wheeler hypothesis by determining whether all martensite plates parallel to the same $\{225\}_{A}$ plane tilt the surface in exactly the same way, or whether there are two different kinds of tilting. This test, together with the comparison of measured angles of tilt with those predicted by the proposed mechanism, was the object of the experimental work which follows.

## Experimental procedure

The $1.35 \%$ carbcn alloy used for the analysis of the martensite relief was prepared from 'Puron' and high purity carbon by melting in a self-sintered alumina crucible in an atmosphere of argon. The ingot was homogenized at $1200^{\circ} \mathrm{C}$. in argon for a total of 3 days, and hot swaged $25 \%$ reduction in diameter. After
machining off the scale produced during the swaging, specimens 0.1 in . in thickness were cut from the rod for use in the experiments.

In order to produce the relief effects in iron-carbon alloys where Ms is above room temperature, it is necessary to prepare the polished surface before heat treatment. After a careful metallographic polish on both sides, the specimens were heated in vacuo at $1200^{\circ} \mathrm{C}$. for 1 hr . and oil quenched. This procedure served to preserve the polished surfaces throughout the annealing, and, after quenching, the martensite plates were clearly visible by virtue of the tilting of the surface which they produce. A photomicrograph of a typical specimen is shown in Fig. 9.

The orientations of the austenite crystals were determined from the traces of not less than two twins in each of two non-parallel surfaces. The austenite twins are clearly visible in surfaces showing the relief effects, and in other surfaces prepared by sectioning and metallographic polishing they can be seen if the surface is lightly etched and examined at low magnification.

The tilting of the surface by a given martensite plate or series of parallel plates was measured using a twocircle optical goniometer. Most of the plates investigated were tilted at angles of $2-5^{\circ}$. The reason for this is that plates with higher angles of tilt are invariably very narrow and consequently difficult to measure. Similarly, the very-low-angle plates with tilts of less than $2^{\circ}$ are also very difficult to measure.

The $\{225\}_{A}$ plane being utilized as habit plane by the particular plate under investigation was determined from the trace of the plate in the surface. Whenever possible a second trace of the plate was determined.

The steps involved in predicting the angle through which a given surface is tilted by the production of a given martensite plate can be summarized as follows:
(1) Prepare a pole figure showing the austenite poles, the pole of the $\{225\}_{A}$ plane which is being utilized as habit plane, and the pole of the tilted surface.
(2) Label the austenite poles so that this $\{225\}_{A}$ plane has the indices $(225)_{A}$, and then plot the standard variant of the Kurdjumow-Sachs relationship.
(3) Determine the indices $(h k l)_{\mathcal{A}}$, of the original austenite surface and from these, using the plane transformation matrix, find the indices, $\left(h^{\prime} k^{\prime} l^{\prime}\right)_{M}$, of the martensite plane into which this surface plane transforms.
(4) Construct the two great circles containing respectively $(225)_{A}$ and $(h k l)_{A}$, and (112) $)_{M}$ and $\left(h^{\prime} k^{\prime} l^{\prime}\right)_{M}$. The intersection of these is the predicted pole of the tilted surface.

## Results

The results obtained for ten of the thirteen martensite plates investigated are shown in Fig. 7. The other three results will be described later. In the construction of this diagram the individual pole figures have all been rotated to bring the austenite into standard projection.

It should be noted that the initial movement of the pole of each surface along a great circle connecting the original pole and the $(225)_{A}$ pole is directly apparent from an examination of the relief effects. The tilted surface contains the trace of the habit plane in the original surface and this restricts the movement of the pole to the great circle referred to above.

In all cases the sense of the tilt was as predicted and the agreement between the predicted and observed angles of tilt is satisfactory. For all martensite plates investigated, the angle between the predicted and observed poles of the tilted surface was less than $3^{\circ}$. In ten of the thirteen cases this angle was less than $2^{\circ}$.


Fig. 7. Comparison of the predicted and observed angles through which polished surfaces are tilted by the production of martensite plates in an iron-carbon alloy containing $1.35 \%$ carbon. O-, Pole of original surface; --, pole of plane $\left(h^{\prime} k^{\prime} l^{\prime}\right)_{M}$ into which the surface plane transforms; $\boldsymbol{e}$, predicted pole of tilted surface; $O$, observed pole of tilted surface.

It should be noted that the angles of tilt are definitely not those which would have been produced had the atom displacements occurred as a simple homogeneous distortion. In this case the predicted pole of the tilted surface would have been the pole $\left(h^{\prime} k^{\prime} l^{\prime}\right)_{M}$ referred to above. In many cases this pole makes angles greater than $10^{\circ}$ with the observed pole of the tilted surface.

As required by the theory, it was observed that all plates parallel to a given $\{225\}_{M}$ plane tilted the surface in exactly the same way. However, it is not known, of course, whether a set of parallel plates did really contain some plates which were twins of the remainder.

Quite a prominent feature of the specimens was the occurrence of grains or portions thereof in which the surface did not appear to be tilted at all, i.e. no martensite plates were visible even though the specimens were fully hardened. Analysis of three such areas revealed
that the predicted angle of tilt was in all cases less than $0.5^{\circ}$. In general the surface will not be tilted at all if the habit plane lies in the surface or if the surface contains the first direction of atom movement. In the three cases referred to above the habit plane was almost in the surface. Cases where thin plates were visible only by virtue of their transverse markings were also seen but no opportunity to analyze such a situation presented itself.* However, it seems quite safe to conclude that in these cases too the surface contained the first direction of motion.

## Discussion

The preceding analysis has led to the formulation of a crystallographic mechanism which is compatible with all the geometrical features of the transformation. Certain paths have been ascribed to the atoms during transformation. These paths specify an intermediate position through which an atom must pass in order that the transformation shall exhibit the observed geometrical characteristics. The actual paths taken in moving to and from this intermediate position are not necessarily the simple straight-line sequences that have been proposed, any more than the path of an atom during twinning in a body-centered cubic metal necessarily consists of a straight-line displacement in the〈111〉 direction. The actual paths in both cases will be determined, as Jaswon \& Wheeler have pointed out, by the potential field in which the atoms move. It should be noted that such an analysis can only describe the atom movements by which the transformation is achieved; it can tell nothing about the simultaneity or otherwise of the movement of different atoms. The mode of development of the martensite plate is therefore still a matter for conjecture.

In view of the very high activation energy which would be required to initiate the simultaneous movement of all the atoms in a plate of austenite, it seems very unlikely that the atomic displacements occur simultaneously. A likely mode of development for the martensite plate is suggested by yet another geometrical feature of the transformation. Examination of the manner in which scratches on a polished surface are bent by the production of martensite plates, shows that the transformation is accompanied by a translation of the austenite on eitner side of the plate, so that the scratch remains continuous across the mariensiteaustenite boundaries (Fig. 10). This behavior suggests the mode of occurrence of the first distortion illustrated in Fig. 8, where successive layers of atoms are added a layer at a time, to a plate nucleus parallel to $(225)_{A}$. As each layer is added the austenite above the developing plate is translated in the $[\overline{112}]_{A}$ direction and therefore remains coherent with the added layer.

Such a mode of development is attractive since each

* The austenite orientation can be determined only in crystals containing two or more twins. Unfortunately this is a relatively rare occurrence.
atom during transformation suffers only two small displacements. The macroscopic component of the first displacement, required by the fact that the first distortion is observed to be homogeneous, occurs while the atom is still part of the austenite. Tbus, according to this picture, the homogeneity of the first distortion is a consequence of the fact that a $(225)_{A}$ plane in which the first displacement has occurred can remain a $(225)_{A}$ plane of the austenite if the austenite is moved in the direction of the first displacement.

The second displacement of any atom will occur immediately after the first, as the intermediate position is unstabie. The two displacements merely represent the path taken by an atom in falling through a potential


Fig. 8. Illustration of the mode of development of a martensite plate. The double circles represent the initial positions of atoms in the $(\mathrm{I} \overline{\mathrm{I}} 0)_{A}$ plane of austenite. As the first displacement occurs successively in the $(225)_{A}$ planes marked 1 to 5 , the austenite above the developirg plate is translated in the $[\text { 112] }]_{A}$ direction to the successive positions indicated by the hollow circles. The fillod circles show the positions of atoms in these $(225)_{A}$ planes after the first displacement has occurred. In the interests of clarity the magnitude of these displacements has been exaggerated, and those (225) A planes which contain atoms in the (110) a plane above the oze shown, have been omitted. The distortion of the row of atoms $A C$ into $A^{\prime} B C$ illustrates the manner in which scratches on a polished surface are bent by the transformation. The second displacement is normel to the plane of this drawing.
gradient. The second displacement occurs in the $[1 \overline{1} 0]_{A} \|[11 \overline{1}]_{M}$ direction, which is perpendicular to the plane of the drawing in Fig. 8. Since the second distortion is a shear on the (ll2) M plane, the displacements suffered by atoms in the interface plane are not all identical as in the first distortion. Therefore, after the second displacement has occurred, the interface plane is no longer common to both austenite and the growing plate, and no translation of the austenite could improve the situation. The second distortion therefore occurs heterogeneously, each atom probably moving to the nearest available equilibrium position. The resulting interface is not coherent in the sense that the interface plane is a plane that is common to both phases. However, such an interface is consistent with the observation


Fig. 9. $1.35 \%$ carbon steel metallographically polished, then heated for 1 hr . at $1200^{\circ} \mathrm{C}$. and oil quenched in vacuo, showing relief effects. $\times 75$.


Fig. 10. Photomicrograph showing the manner in which scratches on a polished surface are bent by the production of martensite plates. Specimen is a $30 \%$ nickel-iron alloy. $\times 150$.
that scratches, which have been bent by the production of martensite plates, remain continuous across the austenite-martensite interface. An interface of the type considered above possesses a kind of coherence since all planes containing the $[1 \overline{1} 0]_{A}$ direction are continuous across the interface.

It is not known at present whether the addition of successive planes occurs one atom at a time, one row at a time, or one complete plane at a time. However, since the $(225)_{A}$ plane is composed of widely separated closepacked rows of atoms, the possibility that the atoms in each close-packed row move essentially simultaneously by the propagation of a compression wave along the row is worthy of attention.

In the above discussion one important feature of the transformation has been neglected, namely, the decrease in atom radius which occurs as a consequence of. the decrease in co-ordination. The transformation as described above would not be accompanied by such a change in atom radius since the $[1 \overline{1} 0]_{A}$ direction transforms without distortion into the $[11 \overline{1}]_{M}$ direction. Thus in order to produce martensite of the correct dimensions an isotropic contraction must occur at some stage or stages of the transformation. The extent of this contraction will depend on the transformation temperature. In pure iron at $910^{\circ} \mathrm{C}$. it would correspond to a $3 \%$ contraction in atom radius and at room temperature to a $1.5 \%$ contraction in atom radius.

Further consideration of the propagating martensite interface indicates the manner in which this contraction will occur. For this interface to migrate by a nechanism which does not involve diffusion it is necessary that the austenite and martensite on either side be strained to conform to the same atomic radius at the interface. If this occurs, then part of the contraction in atom radius takes place in a given plane before the interface reaches this plane, and part after it has passed this plane.

It may be objected that the strain required to permic a coherence of this type would be prohibitive, but this is not necessarily true. In the first place atoms in the neighborhood or the interface do not have a normal co-ordination and hence their equilibrium radii are different from those of atoms located in the austenite and the martensite. Moreover, even if the strain cannot be accommodated elastically there is still the possibility that it can be accommodated by the production of dislocations in the austenite at points remote from the interface. The production of pairs of dislocations of opposite sign would permit bending of the lattice planes in exactly the same way as Hess \& Barrett (1949) have proposed in order to explain the observed bending of the lattice planes on 'kinking'. However, regardless of the manner in which it is achieved, an interface of the type proposed seems to be necessary, both in order to permit
a diffusionless transformation mechanism, and to account for the experimental observation that there is no discontinuity in a scratch when it crosses the austenite-martensite interface.

The simplicity of the crystallographic mechanism of the transformation in pure iron and the low-carbon alloys (less than $1.4 \%$ carbon), probably indicates that we can expect other martensite transformations in pure metals to have equally simple crystallographic mechanisms. However, the presence of alloying elements apparently introduces complications. In $1.7 \%$ carbon steel and in high-nickel steels the mechanism is obviously different from that described above since the martensite orientation may deviate as much as $5^{\circ}$ from the KurdjumowSachs orientation, and the habit plane has approximately the indices $\{259\}_{A}$ instead of $\{225\}_{A}$. These differences are probably to be attributed to the martensite nucleus and the reason for an effect of solute atoms on the orientation of the martensite nucleus is one of the major problems awaiting solution.

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[^0]:    * Present address: C.S.I.R.O., University of Melbourne, Melbourne, Australia.
    $\dagger$ The subscripts $A$ and $M$ are used throughout to indicate indices given relative to austenite and martensite axes, respectively.

[^1]:    * There are twenty-four crystallographically equivalent variants of this relationship. The particular variant considered by Jaswon \& Wheeler is shown in Fig. 1. This variant will hereinafter be referred to as the 'standard variant' of the Kurdjumow-Sachs relationship.

[^2]:    * This result may be interpreted as further confirmation that the Kurdjumow-Sachs relationship does occur in iron-carbon alloys containing less than $1 \cdot 4 \%$ carbon. This is implied in Jaswon \& Wheeler's paper and the present writer agrees with this point of view.

[^3]:    * The tetragonal axis is assumed to be the $[001]_{M}$ axis as predicted by Jaswon \& Wheeler.

